QUANTITATIVE ANALYSIS



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Bridging educators' expertise with learners' ambitions, it delivers accessible, transformative education tailored for today's interconnected world, shaping futures through innovation and excellence

PREFACE

Quantitative analysis is an essential skill for today's business professionals, enabling them to make informed decisions based on data, probability, and mathematical reasoning. This book, **Quantitative Analysis**, is specifically designed to equip CPA and ACCA students with the knowledge, skills, and practical tools necessary to apply quantitative techniques in diverse business contexts.

The content comprehensively covers key topics including mathematical techniques, probability theory, hypothesis testing, regression analysis, time series forecasting, linear programming, and decision theory. Each chapter provides clear explanations, real-world business applications, and a wide range of numerical examples to help students grasp complex concepts effectively.

In addition to theoretical foundations, this book emphasises practical problem-solving and exam preparation. At the end of each chapter, carefully crafted practice questions with detailed solutions enable students to consolidate their learning and build confidence in tackling professional exams.

The approach used in this book reflects the latest CPA and ACCA syllabi requirements. It aims to bridge the gap between theory and practice, supporting learners to become proficient in quantitative methods that enhance decision-making in business operations, finance, procurement, and management.

I trust that this book will serve as a valuable companion throughout your studies and professional journey, helping you master quantitative analysis and excel in your examinations and career.

CPA Ronoh Benson Benhard
Synapse Global Institute
2025

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CHAPTER ONE

MATHEMATICAL TECHNIQUES

1.1 Functions

Learning Objectives

By the end of this section, students should be able to:

- Understand the definition and basic properties of functions.
- Identify different types of functions and their graphical representations.
- Apply functions to solve business-related problems.
- Interpret the real-world applications of functions in business contexts.

1.1.1 Definition of Functions

A function is a mathematical relationship between two sets: a domain (input set) and a codomain (output set). A function assigns each element of the domain to exactly one element of the codomain. In other words, for each input, there is one corresponding output.

Function Notation

A function can be written in the general form:

$$f: X \to Y$$

Where:

- f is the function name.
- X is the domain (set of all possible inputs).
- Y is the codomain (set of all possible outputs).

Mathematically, a function f maps an element x from the domain X to an element $f(x)$ in the codomain Y. This relationship is expressed as:

$$f(x) = y$$

Where:

- x is the input (independent variable).
- $y = f(x)$ is the output (dependent variable).

Functions are widely used in business to model relationships between variables. For instance, a company may use a function to model its revenue (R) based on the number of units sold (Q).

Types of Functions

Functions can take several forms. Some common types include:

1. Linear Functions

A linear function is a function in which the relationship between the input (x) and output (y) is a straight line. Its general form is:

$$y = mx + b$$

Where:

- m is the slope (rate of change).
- b is the y-intercept (value of y when $x = 0$).

Application in Business

Linear functions are often used to model relationships such as cost (C) and output (Q), where the cost increases linearly with production.

2. Quadratic Functions

A quadratic function is a polynomial function of degree 2. Its graph is a parabola, and the general form is:

$$y = ax^2 + bx + c$$

Where:

- a, b, and c are constants.

Application in Business

Quadratic functions are useful for modeling profit maximization, where the relationship between cost, revenue, and profit forms a parabolic curve.

3. Cubic Functions

A cubic function is a polynomial function of degree 3. Its general form is:

$$y = ax^3 + bx^2 + cx + d$$

Where:

- a, b, c, and d are constants.

Application in Business

Cubic functions can model more complex business situations, such as nonlinear supply and demand relationships.

4. Exponential Functions

An exponential function models growth or decay at a constant percentage rate. It is given by:

$$y = a \cdot b^x$$

Where:

- a is the initial value (when $x = 0$),
- b is the growth rate ($b > 0$).

Application in Business

Exponential functions are commonly used in compound interest calculations, population growth, and investment growth.

5. Logarithmic Functions

A logarithmic function is the inverse of an exponential function. It has the form:

$$y = a \cdot log_b(x)$$

Where:

- a is a constant,
- b is the base of the logarithm (usually $b = e$ or $b = 10$),
- x is the independent variable.

Application in Business

Logarithmic functions can model diminishing returns, where growth slows over time.

Graphs of Functions

The graphical representation of a function helps to visually interpret the relationship between the input and output. For example:

- A linear function will have a straight line graph.
- A quadratic function will have a parabolic curve.
- An exponential function will have a J-shaped curve.
- A logarithmic function will have a curve that flattens out as the input increases.

Understanding the graphs of functions is essential for solving business problems, as it allows you to predict the behavior of variables based on changes in input.

Application of Functions in Business

Functions are widely used in business to model various real-world situations. Some key applications include:

1. Cost and Revenue Modelling

Linear functions can model how costs or revenues change with the number of units produced or sold. For example, a company may use a linear function to model total cost, where the cost increases by a fixed amount with each additional unit produced.

2. Profit Maximization

Quadratic functions can be used to model profit, where profit initially increases with sales but then decreases after a certain point due to diminishing returns.

3. Investment Growth

Exponential functions are often used to model the growth of investments over time, where the rate of return is constant, and the investment grows at an accelerating rate.

4. Depreciation of Assets

Logarithmic functions can model the depreciation of assets, where the value of the asset decreases rapidly initially but slows down over time.

Practice Questions and Solutions

Question 1:

A company's revenue function is given by

$$R(x) = 20x + 500,$$

where x is the number of units sold.
(a) Calculate the revenue when 40 units are sold.
(b) If the company sells 100 units, what will the revenue be?
(c) What does the constant 500 represent in the function?

Solution 1:

(a) For $x = 40$:

$$R(40) = 20(40) + 500 = 800 + 500 = 1300.$$

The revenue when 40 units are sold is \$1300.
(b) For $x = 100$:

$$R(100) = 20(100) + 500 = 2000 + 500 = 2500.$$

The revenue when 100 units are sold is \$2500.
(c) The constant 500 represents the fixed revenue, or the revenue the company receives even if no units are sold (i.e., when $x = 0$).

Question 2:

A firm's profit function is given by

$$P(x) = -3x^2 + 150x - 500,$$

where x is the number of units sold.
(a) Calculate the profit when 10 units are sold.
(b) How many units must be sold to achieve a maximum profit?

Solution 2:

(a) For $x = 10$:

$$P(10) = -3(10)^2 + 150(10) - 500 = -300 + 1500 - 500 = 700.$$

The profit when 10 units are sold is \$700.
(b) The maximum profit occurs at the vertex of the parabola, which is given by:

$$x = -\frac{b}{2a}.$$

For this function, $a = -3$ and $b = 150$:

$$x = -\frac{150}{2(-3)} = \frac{150}{6} = 25.$$

Thus, the company must sell 25 units to achieve maximum profit. To find the maximum profit, substitute $x = 25$ back into the profit function:

$$P(25) = -3(25)^2 + 150(25) - 500 = -1875 + 3750 - 500 = 1375.$$

Thus, the maximum profit is $1375, achieved by selling 25 units.

1.1.2 Functions, Equations, Inequalities, and Graphs: Linear, Quadratic, Cubic, Exponential, and Logarithmic Functions

Learning Objectives

By the end of this section, students should be able to:

- Understand the concepts of functions, equations, and inequalities.
- Identify and graph different types of functions, including linear, quadratic, cubic, exponential, and logarithmic functions.
- Apply these functions to solve business-related problems.
- Solve equations and inequalities related to each type of function.
- Interpret the graphical representation of these functions in business contexts.

Functions and Equations

A function relates an input (independent variable) to an output (dependent variable) through a well-defined rule or relationship. A function can be expressed as $f(x) = y$, where x is the input and y is the output.

Equations

An equation is a mathematical statement that asserts the equality of two expressions. It is solved by finding the values of the variables that satisfy the equality.

Inequalities

An inequality is a mathematical statement that compares two expressions, showing that one expression is greater than or less than the other. Inequalities are often used in business to model constraints such as budget limitations or production capacities.

Example in Business:

- **Equation:** $3x + 2 = 14$ (This can be solved to find the value of x).
- **Inequality:** $3x + 2 \leq 14$ (This expresses a constraint where x cannot exceed a certain value).

Types of Functions

In this section, we will discuss the graphical representations, general forms, and applications of five key types of functions: linear, quadratic, cubic, exponential, and logarithmic.

1. Linear Functions

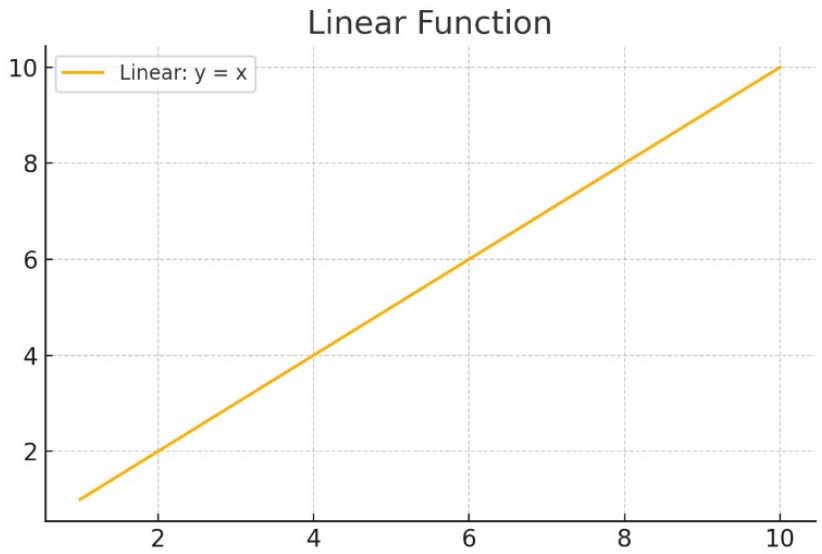
A linear function is a function in which the relationship between the independent variable (x) and the dependent variable (y) is constant. Its graph is a straight line. The general form of a linear function is:

$$y = mx + b$$

Where:

- m is the slope (rate of change of y with respect to x),
- b is the y-intercept (the value of y when $x = 0$).

Graph of a Linear Function



The graph of a linear function is a straight line, and the slope determines the steepness of the line:

- If $m > 0$, the line rises from left to right.
- If , the line falls from left to right.

Application in Business

Linear functions are commonly used to model cost and revenue. For example, if a company sells a product for £10 per unit, the revenue (R) from selling x units can be modeled as:

$$R(x) = 10x.$$

This means that the revenue increases by £10 for every unit sold.

2. Quadratic Functions

A quadratic function is a polynomial function of degree 2. Its graph is a parabola and can be written in the form:

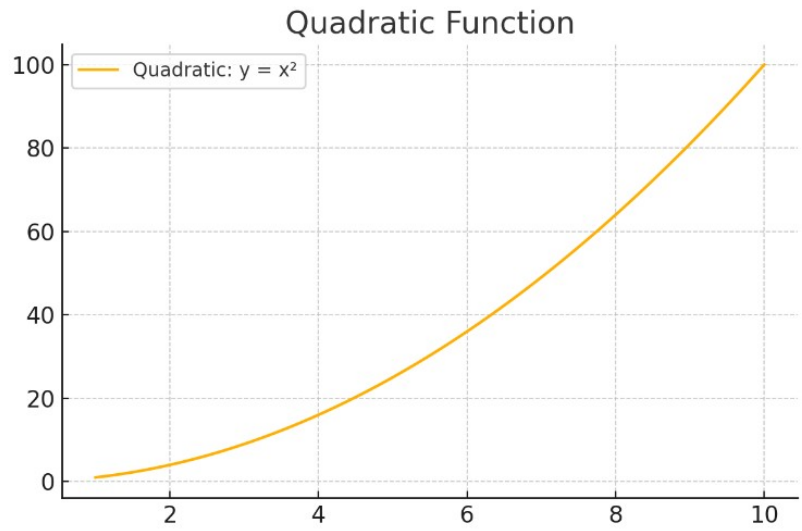
$$y = ax^2 + bx + c$$

Where:

- a, b, and c are constants.

The value of a determines whether the parabola opens upwards ($a > 0$) or downwards ().

Graph of a Quadratic Function



The graph of a quadratic function is a parabola. The vertex represents the maximum or minimum value, depending on the sign of a.

Application in Business

Quadratic functions are useful in business for modeling profit maximization. For example, a company's profit might be given by a quadratic function like:

$$P(x) = -2x^2 + 40x - 200.$$

This function could model profit where x is the number of units sold, and the maximum profit occurs at the vertex of the parabola.

3. Cubic Functions

A cubic function is a polynomial function of degree 3. Its general form is:

$$y = ax^3 + bx^2 + cx + d$$

Where:

- a, b, c, and d are constants.

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