QUANTITATIVE ANALYSIS

Applied Business Mathematics, Statistics, Forecasting and Decision Science

SYNAPSE
GLOBAL INSTITUTE

Connecting Minds. Transforming Futures.

QUANTITATIVE ANALYSIS

Applied Business Mathematics, Statistics, Forecasting and Decision Science

A comprehensive professional study and reference book

SYNAPSE GLOBAL INSTITUTE

Connecting Minds. Transforming Futures.

Copyright and Publication Information

First Edition, 2026

Copyright © 2026 Synapse Global Institute. All rights reserved.

Published by Synapse Global Institute.

ISBN: To be assigned.

No part of this publication may be reproduced, stored in a retrieval system, transmitted, distributed or adapted in any form or by any means without prior written permission from the publisher, except for brief quotations used for review, scholarship or other uses permitted by law.

This publication is designed for education, professional study and applied quantitative analysis. Worked examples, business cases and assessment-style questions are provided for learning purposes. Readers should apply professional judgement and verify assumptions, data and computational outputs when using quantitative methods in live business decisions.

Product names, organisations and scenarios used in examples may be illustrative unless expressly identified otherwise.

Preface

Quantitative analysis is most useful when it helps a decision maker move from information to a defensible conclusion. This book therefore treats mathematics, statistics and operations research as practical tools for business rather than as isolated techniques.

The book progresses from functions and matrix algebra through calculus, descriptive statistics, probability, statistical inference, regression, forecasting, optimisation and decision theory. Each chapter combines concise conceptual explanation with worked examples, structured calculation methods, interpretation guidance and practice designed to strengthen both understanding and professional presentation.

A recurring editorial principle throughout the book is that a numerical result is rarely the end of the analysis. Strong quantitative work identifies the method, states assumptions where relevant, shows a checkable calculation path, attaches correct units and explains what the result means for the business problem.

The final chapter consolidates the full toolkit through mixed problems and an integrated mock examination so that the reader practises method selection without relying on topic labels.

How to Use This Book

Start with the chapter overview. Use the chapter map and learning outcomes to identify the techniques and decisions covered.

Follow worked examples actively. Pause before each solution, attempt the setup independently, then compare the method and interpretation.

Write complete workings. State the formula or model, substitute values in a logical order, retain sufficient intermediate accuracy and attach units.

Interpret every important result. Translate coefficients, probabilities, test decisions, forecasts and optimisation outputs into concise business language.

Use assessment sections as timed practice. Attempt questions under realistic time pressure before consulting worked solutions or answer checkpoints.

Maintain a formula and error log. Record methods you confuse, arithmetic errors you repeat and assumptions you forget; revisit them during consolidation.

Mathematical and Statistical Notation

The book uses conventional mathematical notation throughout. Powers are written as superscripts, matrices are arranged in aligned rows and columns, probability statements are expressed symbolically, and statistical quantities use standard notation. Programming-style shorthand is avoided in learner-facing mathematics.

Concept	Preferred notation	Meaning / use
Powers	x^2, x^3, e^x	Use superscripts rather than caret notation.
Derivative	dy/dx, $f'(x)$, d^2y/dx^2	Rate of change and higher-order derivatives.
Integral	$\int f(x)\,dx$	Accumulation or recovery of a total function.
Mean	$\bar{x}$	Arithmetic mean of a sample or dataset.
Variance and standard deviation	σ^2, σ or s^2, s	Measures of dispersion.
Probability	$P(A)$, $P(A \mid B)$	Unconditional and conditional probability.
Set operations	$A \cup B$, $A \cap B$, A'	Union, intersection and complement.
Correlation	r or r_s	Pearson or Spearman correlation coefficient.
Linear programming	Max Z or Min Z	Objective function subject to constraints.
Expected value	E(X), EMV	Probability-weighted expected outcome.

Contents

PART I | MATHEMATICAL FOUNDATIONS 1
Functions and Matrix Algebra 1
Calculus for Business Decisions: Differentiation and Integration 24
PART II | STATISTICS, PROBABILITY AND INFERENCE 46
Descriptive Statistics and Distribution Shape 46
Set Theory and Probability Theory 64
Probability Distributions and Business Applications 79
Hypothesis Testing and Estimation 93
PART III | MODELLING, FORECASTING AND OPTIMISATION 106
Correlation and Regression Analysis 106
Time Series Analysis and Forecasting 117
Linear Programming: Formulation, Graphical and Simplex Methods 131
PART IV | DECISION ANALYSIS AND CONSOLIDATION 142
Decision Theory under Certainty, Risk and Uncertainty 142
Integrated Consolidation and Mock Examination 154

PART I | MATHEMATICAL FOUNDATIONS

CHAPTER 1

Functions and Matrix Algebra

Build fluency in functions, equations, inequalities, graphs and matrix algebra; then apply the techniques to revenue, cost, demand, depreciation, resource planning, simultaneous equations, Markov transitions and input-output relationships.

Chapter Overview

This chapter is designed for the opening chapter of Quantitative Analysis. It concentrates on two foundations that repeatedly support later topics: mathematical functions and matrix algebra. The aim is not merely to memorise notation. The learner should be able to translate a business statement into a mathematical relationship, select a suitable method, carry out the calculation cleanly, and interpret the result in business language.

The chapter is deliberately exam-focused. Recent examinations repeatedly assess definition and interpretation of functions and matrices, construction of linear and quadratic relationships from observed data, practical use of revenue, cost and demand functions, matrix addition and multiplication, solving matrix equations, and the use of matrices in operational settings. The strongest answers therefore combine correct method selection, orderly workings, units and a brief interpretation.

> **Important examination-format note**
>
> Professional examinations may use either compulsory-question or selective-question structures. Preparation should therefore emphasise broad syllabus coverage, method selection and the ability to show complete workings across all major topics.

Chapter Map

Area	Coverage	What you must be able to do
Functions	Definition; equations; inequalities; graphs; linear, quadratic, cubic, exponential and logarithmic functions	Recognise, formulate, evaluate, graph and interpret business relationships.
Business applications	Revenue, cost, profit, demand, depreciation, growth and other functional relationships	Turn narrative and observed data into equations and explain the result.
Matrix algebra	Definition; matrix types; addition, subtraction, multiplication, transposition and inversion up to 3 × 3	Carry out operations accurately and check conformability and determinant conditions.
Matrix applications	General business modelling, statistical modelling, Markov analysis and input-output analysis	Set up matrix models, perform required multiplication or inversion, and interpret each result.

Learning outcomes

- Define a function, domain, range, equation, inequality and matrix using precise quantitative language.
- Distinguish linear, quadratic, cubic, exponential and logarithmic functions and recognise their characteristic graphs.

- Derive linear functions from two observations and quadratic functions from three observations.
- Apply functions to demand, revenue, cost, profit, depreciation, growth and capacity situations.
- Perform matrix addition, subtraction, scalar multiplication, multiplication and transposition.
- Calculate determinants and inverses of square matrices, including matrices of order 3 × 3.
- Use matrix algebra to solve simultaneous equations and matrix equations such as AX = B.
- Use matrices to model production-resource requirements, input-output relationships and one-step Markov transitions.
- Present professional examination-style workings clearly, with correct notation, units and interpretation.

Synapse mathematical presentation standard

Mathematics in this chapter is written in conventional professional notation. Powers are shown as superscripts, for example x^2 and x^3, rather than with programming-style caret notation. Matrix inverses are written A^{-1}. Multiplication is shown with a multiplication sign, a centred dot where useful, or by brackets. Fractions and ratios are laid out clearly, and each symbol is defined before use. The same standard should be followed in classwork and examination answers.

Use	Preferred presentation	Avoid in handwritten/exam work
Powers	x^2, Q^3, e^{kt}	Programming-style exponent notation
Inverse	A^{-1}	Programming-style inverse notation
Multiplication	3Q, 4 × 5, A B	Ambiguous adjacent numbers
Equality	P = 105 − 0.10Q	Chained arithmetic without equals signs
Matrices	Bracketed rows and columns with stated order	Unlabelled arrays of numbers
Final result	Sh. 920,000; 500 units; 38.25%	A bare number with no unit

Assessment focus and recurring question patterns

Recurring task	Typical command verb	Core skill
Explain or define a term	Explain / Define / Distinguish	Give precise meaning and, where useful, a short business context.
Derive a linear function	Formulate / Determine	Use two observations to obtain slope and intercept.
Derive a quadratic function	Formulate / Determine	Set up three simultaneous equations and solve coefficients.
Apply a business function	Calculate / Determine / Interpret	Substitute correctly, solve for the required variable and retain units.
Matrix operations	Determine / Calculate	Check order and conformability before adding or multiplying.
Matrix equation	Find / Solve	Use $A^{-1}B$ where the inverse exists, or solve equivalent simultaneous equations.
Applied matrix model	Using matrix algebra, calculate	Build quantity, resource or cost matrices and interpret every resulting entry.

PART A | FUNCTIONS, EQUATIONS, INEQUALITIES AND GRAPHS

Understanding functions

A function is a rule that associates every permissible input with exactly one output. If the input variable is x and the output is y, the relationship may be written y = f(x). The notation f(x) is read as "f of x". It does not mean f multiplied by x. In business, the input may be units produced, time, advertising expenditure, labour hours or selling price; the output may be cost, revenue, demand, asset value or profit.

Key definition

The domain is the set of input values for which a function is defined. The range is the set of output values generated by the function. Business conditions often restrict the mathematical domain: output cannot be negative, capacity may impose an upper bound, and a logarithmic function requires a positive argument.

Function notation $y = f(x)$

x is the independent variable; y is the dependent variable.

WORKED EXAMPLE Function evaluation and business domain

A delivery company estimates daily operating cost by C(q) = 18,000 + 240q, where q is the number of deliveries completed per day and the fleet can handle at most 160 deliveries.

- For 75 deliveries, substitute q = 75.

Evaluation $C(75) = 18{,}000 + 240(75) = 36{,}000$

- The economically relevant domain is $0 \leq q \leq 160$, assuming q is measured in whole deliveries.

Answer checkpoint

The estimated daily operating cost at 75 deliveries is Sh. 36,000. The relevant business domain is from 0 to 160 deliveries.

Exam technique

Always distinguish the mathematical formula from the practical domain imposed by the scenario.

Functions, equations and inequalities

A function describes a relationship. An equation states that two expressions are equal and is often solved to find an unknown. An inequality compares expressions using <, >, ≤ or ≥ and is commonly used to represent business constraints. For example, a capacity restriction may be written 3x + 2y ≤ 120, while a minimum service requirement may be written x + y ≥ 40.

Concept	Meaning	Business illustration
Function	A rule mapping an input to one output	C(q) = 18,000 + 240q
Equation	A statement of equality	18,000 + 240q = 42,000

Concept	Meaning	Business illustration
Inequality	A statement of order or constraint	$240q \leq 38{,}400$
Graph	A visual representation of a relationship	Cost plotted against number of deliveries

WORKED EXAMPLE Solving a business inequality

A branch has a weekly overtime budget of Sh. 72,000. Each additional labour hour costs Sh. 1,200. Determine the maximum number of overtime hours the branch can authorise.

Constraint $1{,}200h \leq 72{,}000$

- Divide both sides by 1,200. Because the divisor is positive, the direction of the inequality does not change.

Solution $h \leq 60$

Answer checkpoint

The branch may authorise at most 60 overtime hours.

Exam technique

If you multiply or divide an inequality by a negative number, reverse the inequality sign.

Linear functions

A linear function has a constant rate of change. Its graph is a straight line. The general form is y = a + bx, where a is the intercept and b is the slope or gradient. A positive slope indicates that y increases as x increases; a negative slope indicates that y falls as x increases.

Linear function $y = a + bx$

Slope from two points $m = (y_2 - y_1) \div (x_2 - x_1)$

Once the slope is known, substitute either observed point into y = a + bx to determine the intercept. This two-step approach is one of the most important chapter-one techniques because it appears in demand, revenue, cost and depreciation modelling.

WORKED EXAMPLE Deriving a linear depreciation function

A machine is valued at Sh. 1,520,000 after two years and Sh. 920,000 after five years. Assume that value declines linearly with time. Derive the value function V(t), where t is the number of years after purchase.

Slope $m = (920{,}000 - 1{,}520{,}000) \div (5 - 2) = -200{,}000$

- The machine loses Sh. 200,000 in value per year.

Model $V(t) = a - 200{,}000t$

- Use t = 2 and V = 1,520,000 to determine a.

Intercept $1{,}520{,}000 = a - 400{,}000 \Rightarrow a = 1{,}920{,}000$

Value function $V(t) = 1{,}920{,}000 - 200{,}000t$

Answer checkpoint

V(t) = 1,920,000 − 200,000t. The intercept represents the estimated acquisition value at t = 0.

Exam technique

State what the slope means. Here, the negative slope is annual depreciation of Sh. 200,000.

WORKED EXAMPLE Linear demand, revenue and profit relationships

Market observations show that a product can be sold at Sh. 75 when quantity demanded is 300 units, and at Sh. 55 when quantity demanded is 500 units. Total cost is C(Q) = 5,000 + 45Q.

Demand slope $m = (55 - 75) \div (500 - 300) = -0.10$

Demand form $P = a - 0.10Q$

Demand intercept $75 = a - 0.10(300) \Rightarrow a = 105$

Demand function $P = 105 - 0.10Q$

Revenue function $R(Q) = P \times Q = (105 - 0.10Q)Q = 105Q - 0.10Q^2$

Profit function $\pi(Q) = R(Q) - C(Q) = -0.10Q^2 + 60Q - 5{,}000$

- The break-even quantities satisfy $\pi(Q) = 0$.

Break-even equation $-0.10Q^2 + 60Q - 5{,}000 = 0$

Factorised form $Q^2 - 600Q + 50{,}000 = 0 = (Q - 100)(Q - 500)$

Continue learning with SGI

You have reached the end of this sample: the first 13 pages of the handbook.

Purchase the full handbook to continue reading and access all chapters.

Purchase the full handbook

Synapse Global Institute

Connecting Minds. Transforming Futures.

www.sgi.ac